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The Bilinear Formula in Soliton Theory of Optical Fibers

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ABSTRACT

Solitons are wave phenomena or pulses that can maintain their shape stability when propagating in a medium. In optical fibers, they become general solutions of the Non-Linear Schrödinger Equation (NLSE). Despite its mathematical complexity, NLSE has been an interesting issue. Soliton analysis and mathematical techniques to solve problems of the equation keep doing. In this paper, we review the form of the bilinear formula for the case. We re-observed a one-soliton solution, their stability, and like soliton trains based on the formula, also verified the work of the last researcher. Here, the mathematical parameters of position $\alpha^{(0)}$ and phase η are verified to become features of change in horizontal position and phase of one soliton in the (z, t) plane during propagation. In addition, we notice the soliton has established stability. Finally, for the condition Kerr effect focusing or the group velocity dispersion β_2 more dominates, we present like the soliton trains in optical fibers under modulation instability of plane wave.

I. INTRODUCTION

Technology has become a device and a primary human need in the modern world of communication. Reducing loss and improving the transmission quality of the technological communication system is the greatest challenge. The utilization of optical fibers is one of the alternative questions to answer. However, in modern optical fiber communication, loss, transmission quality, and information capacity issues have increasingly become a concern (Yan, X. W. & Chen, 2022). Hence, we need an idea that supports the optimal and efficient control of the information transmission process. It is the offering to process the transmission of information in the form of soliton pulses (Liu, W. J., Tian, B., Zhang, H. Q., Li, L. L. & Xue, 2008).

Hasegawa, A. & Tappert (1973) was the first scientist to present soliton pulses to reduce loss issues in optical fiber transmission of information. It is a theoretical concept regarding modeling an optical beam (information carrier) into a single wave packet (soliton pulses) stable in fibers during propagation. The solitons theory in optical fiber has attracted considerable attention (Chen, X., Sun, Y., Gao, Y., Yan, X., Zhang, X., Wang, F., Suzuki, T., Ohishi, Y. & Cheng, 2021; Ripai, A., Abdullah, Z., Syafwan, M. & Hidayat, 2020; Yan, X. W. & Chen, 2022). Theoretically, we know fiber solitons dynamics obtainable formulated by the Non-Linear Schrödinger Equation (NLSE) (Agrawal, 2013). Its stability is clear from the right balance between the nonlinearity and the dispersion properties of optical fibers (Agrawal, 2013; Ripai, A., Abdullah, Z., Syafwan, M. & Hidayat, 2020). Only second-order dispersion is familiar considered in the equation yet. It urges our understanding of the bright soliton existence in the anomalous dispersion regime and dark solitons (the intensity profile contains a dip in a uniform background) in a normal dispersion regime (Mei-Hua, L., You-Shen, X. & Ji, 2004). When short pulses are considered (to nearly 50 fs), third-order dispersion becomes essential, so it must include in the NLSE model. Then, as the pulse width becomes even narrower (below 10 fs), the fourth-order dispersion must also be considered (Palacios, S. L. & Fernández-Diaz, 2001).

Hence, (Mei-Hua, L., You-Shen, X. & Ji, 2004) were motivated to study the extent to the higher order by considering the generalized NLSE with third and fourth-order dispersion and cubic-quintic nonlinearity.

We rewrite the model as follows:

$$i\psi_z - \frac{\beta_2}{2}\psi_{tt} - i\frac{\beta_3}{6}\psi_{ttt} - \frac{\beta_4}{24}\psi_{tttt} + \gamma_1|\psi|^2\psi + \gamma_2|\psi|^4\psi = 0 \tag{1}$$

where $\psi(z, t)$ denotes the complex amplitude of the optical pulse envelope, and β_k, (k = 2, 3, 4) denotes the coefficients of k-th order dispersions, respectively. $\gamma_{1,2}$ are the coefficients of the cubic and quintic nonlinear terms, respectively. The second term is called the group velocity dispersion, and the fifth term is the Kerr effect (or cubic nonlinearity). Meanwhile, the other terms are the expansion of the dispersion to the third and fourth-order and the quintic nonlinearity (Mei-Hua, L., You-Shen, X. & Ji, 2004).

Equation (1) is the extension of general NLSE models into high order: (a) if $\beta_3 = \beta_4 = \gamma_2 = 0$, it is standard NLSE and is known to describe the fundamental solitons in optical fibers (pulse width above 100 fs), (b) if $\beta_3 = \beta_4 = 0$, i.e., no third and fourth-order dispersions, we can predict the bright-dark solitons even in the general dispersion regime, and only the situation of $\beta_3 = 0$ satisfies this result, (c) if $\gamma_2 = 0$, i.e., no quintic nonlinear term, it pushes our understanding of problems related to the time behavior of amplitude, velocity, and modulation instability of optical solitons (Yan, X. W. & Chen, 2022). We can understand the soliton existence in optical fibers from the solution of Eq. (1). (Huang, Y. & Liu, 2014) justifies a direct transformation to derive the envelope wave solutions. (Sultan, A. M., Lu. D., Arshad, M., Rehman, H. U & Saleem, 2020) constructed the simple soliton solutions by the exponential expansion method. On the other hand, (Hong, 2002; Ma, W.-X. & Zhou, 2018; Mei-Hua, L., You-Shen, X. & Ji, 2004; Su, J.-J. & Gao, 2017) also obtained new optical solitons from this model. Recently, based on the bilinear formula, (Yan, X. W. & Chen, 2022) first studied the interaction of two solitons placed close together in optical fibers from Eq. (1). This paper is the bilinear formula perspective in the soliton theory of optical fibers through the equation. We re-observed the solution for a one-soliton from the formula and verified the work of (Yan, X. W. & Chen, 2022). As a novelty, we show the stability of its soliton during propagation. Besides, we investigate like soliton trains due to it the modulation instability of the system. We present and analyze based on the principles and related physical parameters.

II. BILINEAR FORMULA

Here, we rewrite bilinear formula to obtain soliton solutions of Eq (1). Following (Yan, X. W. & Chen, 2022), let us consider the transformation function of the complex amplitude of optical beams envelope:

$$\psi = \frac{y(z,t)}{f(z,t)}, \tag{2}$$

where $y(z, t)$ is a complex function and $f(z, t)$ is a real function. In addition, we introduce a complex auxiliary function $g(z,t)$. Substituting Eq. (2) in (1), we arrive at

$$\begin{aligned}
&i\frac{f\,y_z - y\,f_z}{f^2} - \frac{1}{2}\beta_2\left(\frac{f\,y_{tt} - 2y_t f_t + y\,f_{tt}}{f^2} + \frac{2y\,f_t^2 - 2y\,f\,f_{tt}}{f^3}\right) \\
&-\frac{\beta_3}{6i}\left(\frac{f\,y_{ttt} - 3y_{tt} f_t + 3y_t\,f_{tt} - y\,f_{ttt}}{f^2} + \frac{6y_t f_t^2 - 6y_t f\,f_{tt}}{f^3} + \frac{6y\,f\,f_t\,f_{tt} - 6y\,f_t^3}{f^4}\right) \\
&-\frac{1}{24}\beta 4\left(\frac{f\,y_{tttt} - 4y_{ttt}\,f_t + 6y_{tt}\,f_{tt} - 4y_t\,f_{ttt} + y\,f_{tttt}}{f^2} + \frac{12y_{tt}\,f_t^2 - 12y_{tt}\,f_{ttt}\,f}{f^3}\right. \\
&+\frac{24y_t\,f_t\,f_{tt}\,f - 24y_t f_t^3 + 6y\,f_{tt}^2\,f - 6y\,f_t^2\,f_{tt} + 6y\,f_{tt}^2\,f - 6y\,f_t^2\,f_{tt}}{f^4} \\
&\left.+\frac{-2y\,f_{tttt}\,f + 8y\,f_t\,f_{ttt} - 6y\,f_{tt}^2}{f^3} + \frac{24y\,f_t^4 - 24y\,f_t^2\,f_{tt}\,f}{f^5}\right) + \frac{\gamma_2 y^3 y^{*2}}{f^5} + \frac{\gamma_1 y^2 y^*}{f^3} = 0.
\end{aligned} \tag{3}$$

Then, by the use of the condition $\gamma_1 = -\beta_2$ and $\gamma_2 = \beta_4$, we can eliminate the last two terms in Eq. (3).

Finally, we can set the bilinear formula for Eq. (1):

$$\frac{i\beta_3}{2}\left(y_t f^2 D_t^2 y\, f - y\, f_t D_t^2 ff\right) + \frac{\beta_4}{4}\left(D_t^2 y\, f \cdot D_t^2 ff + \frac{1}{6}\left(y\, f - f^2\right) D_t^4 ff\right) = gf^3, \tag{4}$$

$$\left(-iD_z + \frac{1}{2}\beta_2 D_t^2 + \frac{1}{6}i\beta_3 D_t^3 + \frac{1}{24}\beta_4 D_t^4\right) yf = gf,\; D_t^2 ff = 2yy^*, \tag{5}$$

where D_z, D_t are the bilinear operators, and * denotes complex conjugate (Hirota, 2004; Yan, X. W. & Chen, 2022).

III. ONE-SOLITON SOLUTION

We can complete the soliton solutions of Eq. (1) by expanding functions of f, y, and g in bilinear formula (4) and (5) as the expression of δ. In detail, the reads as (Yan, X. W. & Chen, 2022):

$$f = 1 + f^{(2)}\delta^2 + f^{(4)}\delta^4 + f^{(6)}\delta^6 + \cdots, \tag{6}$$

$$y = y^{(1)}\delta + y^{(3)}\delta^3 + y^{(5)}\delta^5 + \cdots, \tag{7}$$

$$g = g^{(0)} + g^{(2)}\delta^2 + g^{(4)}\delta^4 + g^{(6)}\delta^6 + \cdots, \tag{8}$$

where $f^{(l)}$, $l = 2, 4, 6, \ldots$ denote the real functions. Meanwhile, $y^{(m)}$, $m = 1, 3, 5, \ldots$ and $g^{(n)}$, $n = 0, 2, 4, 6, \ldots$ are complex ones.

Take the definition of $y = y^{(1)}\delta$, $f = 1 + f^{(2)}\delta^2$, and $g = g^{(0)}$ in Eq. (6)–(8). After that, substituting into the bilinear formula (4) and (5) (Yan, X. W. & Chen, 2022). In this case, we can obtain the one-soliton solution in the form (supposing $\delta = 1$)

$$\psi = \frac{y^{(1)}}{1 + f^{(2)}}, \tag{9}$$

with $y^{(1)} = e^{\alpha}$, $f^{(2)} = \dfrac{e^{\alpha+\alpha^*}}{(\eta+\eta^*)^2}$, $g^{(0)} = 0$, and

$$\alpha = \eta t - \left(\frac{1}{2}i\beta_2\eta^2 - \frac{1}{6}\beta_3\eta^3 + \frac{1}{24}i\beta_4\eta^4\right) z + \alpha^{(0)}, \tag{10}$$

where η and $\alpha^{(0)}$ denote the complex constants (Yan, X. W. & Chen, 2022).

IV. RESULT AND DISCUSSION

In this situation, we trace the one-soliton solution in optical fibers based on the extension of general NLSE (Eq. 1) models with the solution given by Eq. (9). Under the suitable parameter setting, Eq. (9) provides a unique description of the theoretical existence of one-soliton along in the (z, t) plane (Figure (1)-(3)):

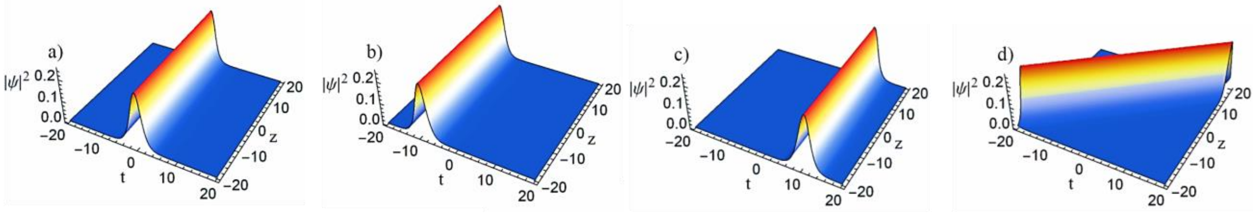

Figure 1 One-soliton of solution (9) by choosing suitable parameters $\beta_2 = 0.02$, $\beta_3 = 0.5$, and $\beta_4 = 0.2$: (a) $\eta = \alpha^{(0)} = 0.5$; (b) $\eta = 0.5$, $\alpha^{(0)} = 5$; (c) $\eta = 0.5$, $\alpha^{(0)} = -5$; (d) $\eta = 0.5 + 2i$, $\alpha^{(0)} = 0.5$.

Figure (1a) shows us how the one-soliton propagates in the (z, t) plane. In this case, we see the position and phase of the soliton change based on the setting of the $\alpha^{(0)}$ and η values. If we increase the value of $\alpha^{(0)}$, one-soliton moves horizontally along the negative half axis of t (Figure (1b)). Contrary, if we decrease the value of $\alpha^{(0)}$, one-soliton moves horizontally along the positive half axis of t (Figure (1c)). By choosing $\eta = 0.5 + 2i$, Figure (1d) shows that the phase of one-soliton changes. Overall, soliton profiles in Figure (1a)–(1d) are not visible to change. In a more physical sense, it maintains its shape stability during propagation. We present their stability verification in Figure (2) qualitatively. Therefore, we can justify (Yan, X. W. & Chen, 2022) claim that only features $\alpha^{(0)}$ and η affect the horizontal position and phase one-soliton in the (z, t) plane.

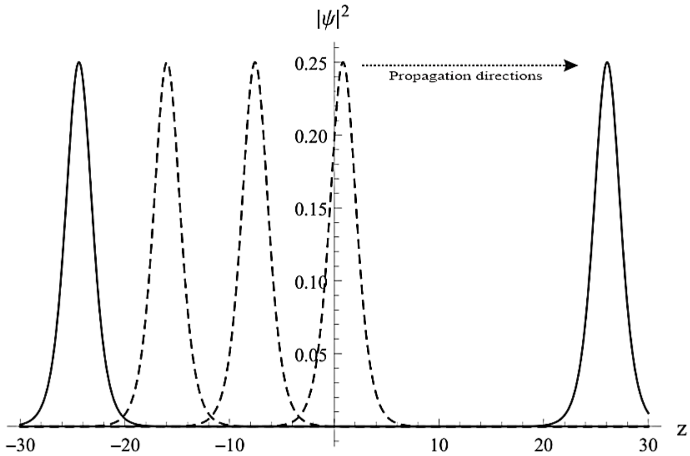

Figure 2 One-soliton propagation in the (z, t) plane by choosing parameters: $\beta_2 = 0.02$, $\beta_3 = 0.5$, $\beta_4 = 0.2$, $\eta = 0.5 + 2i$ and $\alpha^{(0)} = 0.5$.

Figure (2) shows the schematics of one-soliton propagation in the (z, t) plane. The one-soliton appears to have well-established stability during propagation along the axis of z. Corresponding to the shape and the peak intensity of pulses, we can verify that there does not evolve along the plane. The stability of one-soliton is clear from the right balance between nonlinearity and optical fiber dispersion (Agrawal, 2013; Ripai, A., Abdullah, Z., Syafwan, M. & Hidayat, 2020). Hence, we need to set the suitable dispersion parameters of β_2, β_3, and β_4, as given in Figure (2).

Suppose we set the value of the third and fourth-order dispersion parameters (β_3 and β_4) to be smaller. The NLSE (Eq. 1) models will have a more dominant consequence on the group velocity dispersion of β_2. Its demands cubic nonlinearity (or Kerr effect) to take on more than quintic nonlinearity in one-soliton stabilization. Thus, we know that the Kerr effect type balances the group velocity dispersion to achieve soliton pulse stability (Agrawal, 2013).

Meanwhile, quintic nonlinearity is more to the third and fourth-order dispersion (Baizakov, B. B., Bouketir, A., Al-Marzoug, S. M. & Bahlouli, 2019). In a more technical sense, it assumes the Kerr effect's focusing, where the support of quintic nonlinear is considered weak or even negligible. The consequence of setting smaller values of β_3 and β_4 demand focusing on the Kerr effect in optical fibers. This case pushes our understanding of the modulation instability of one-soliton pulse along the (z, t) plane. The form of modulation instability is like the soliton trains phenomenon in the plane (Figures (3b) and (3c)).

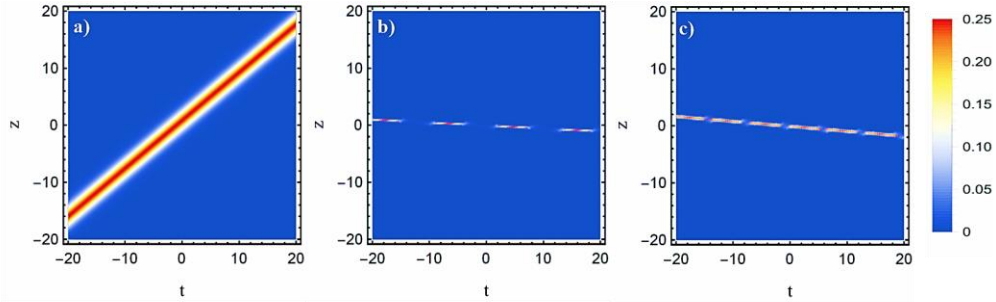

Figure 3 (a) one-soliton stability in the (z, t) plane by choosing values $\beta_2 = 0.02$, $\beta_3 = 0.5$, $\beta_4 = 0.2$, $\eta = 0.5 + 2i$ and $\alpha^{(0)} = 0.5$, also, modulation instability when we choose (b) $\beta_3 = -5$ and (c) $\beta_4 = -8$.

Figure (3) shows the modulation instability of plane waves (soliton pulses) in optical fibers. First, we see that the one-soliton is stable after choosing suitable parameters (Fig. (3a)). Then, we set the parameter values as in the figure. In this situation, we observed modulation instability that assumes the focusing of Kerr effects, i.e., the group velocity dispersion β_2 takes more dominants. Of course, this situation indicates that the one-soliton is in anomalous regimes (Agrawal, 2013; Ripai, A., Abdullah, Z., Syafwan, M. & Hidayat, 2021; Ripai, A. Sutantyo, T. E. P., Abdullah, Z., Syafwan, M. & Hidayat, 2021). We found that the modulation instability causes the one-soliton to propagate like soliton trains along the plane (Figures (3b) and (3c)). It confirms the correctness of soliton train theories in optical fibers supported by a modulation instability of plane wave, as did (Baizakov, B. B., Bouketir, A., Al-Marzoug, S. M. & Bahlouli, 2019). Finally, soliton trains are solitons that appear to be series in a row. Here, they present from modulation instability support, as shown in Figures (3b) and (3c).

V. CONCLUSION

In this paper, we understand that the extension of general NLSE (1) models into the third and fourth-order dispersions and quintic nonlinearity traced using the bilinear formula presents the one-soliton solution with established stability in optical fibers. We verified that the mathematical parameters of $\alpha^{(0)}$ and η affect one-soliton horizontal position and phase during propagation in the plane. Finally, we found like the soliton trains in the plane under modulation instability of plane wave, i.e., for the condition that assuming Kerr effect focusing or the group velocity dispersion more dominates.

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